

Exploiting Gossen's second law: a simple proof of the Euler equation and the maximum principle

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Abstract: We offer simple and intuitive proofs of the Euler equation and the maximum principle based on Gossen's Second Law, one of the best known results in economics.

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1. Introduction

Gossen's Second Law, literally stating that the marginal utility of one extra dollar (MUD) spent on each consumption good is the same for all the consumption goods as required by budget-constrained utility maximization, is unquestionably one of the best known results to students in economics. On the other hand, in the inter-temporal decision context, Euler's equation proves to be a far more powerful tool, from which one can readily obtain Gossen's second law in its inter-temporal version wherein the same goods (or services) at different times are formally viewed as different goods defined by the date and hence MUD remains the same across time. This short article aims to show that one can indeed *reverse* the reasoning, making use of Gossen's second law to prove the Euler equation without resorting to the calculus of variations. (The proof of the Euler equation using the calculus of variations is found in almost any textbook in mathematical economics, see e.g., Lancaster 1987, pp.377-9 and Léonard and Van Long 1995, pp.170-1). Furthermore, by similar argument, the maximum principle can also be established. Our approach has an obvious advantage: it is essentially based on one basic result found in any textbook on intermediate microeconomics, suggesting that, in addition to the familiar exercises of establishing theorems/propositions in economics by using mathematical reasoning, economic intuition may sometimes help establish theorems in mathematics as well. Serving as a nice example, Gossen's

