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**COMPETITION AND ACCESS REGULATION IN THE TELECOMMUNICATIONS
INDUSTRY WITH MULTIPLE NETWORKS**

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Abstract:

We develop a framework, extending the conventional duopoly model by replacing the Hotelling line with a simplex in high-dimension spaces, to study the competition and access regulation of multiple networks. We first characterize the competitive equilibrium when the substitutabilities of the networks are not too high, or the access charges are nearly cost-based. We then analyze how the equilibrium market shares respond to marginal variations in the access charges under various regimes of access regulation, and thereby examine the efficiency implications of such regulation regimes. In particular, we analyze the asymmetric scenario in which some networks are incumbent and some are entrants. It is shown that some existing results of the duopoly do not extend to a multi-firm setting, largely because regulation of multiple networks is structurally far richer. (*JEL*: L96, L51, D43)

Keywords: Telecommunications, oligopoly, network competition, access regulation.

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Competition and Access Regulation in the Telecommunications Industry with Multiple Networks

1. Introduction

The purpose of this study is to investigate how price competition between multiple networks results in equilibrium, and how access charge regulation may be adjusted so as to promote competition and thereby improve efficiency when there are multiple firms competing in the market. Over the past decade, the study of oligopoly competition in the telecommunications industry has been almost routinely confined to the Hotelling model of duopoly that was first introduced by Laffont, Rey and Tirole (1998a) and Armstrong (1998).³ We extend this canonical Hotelling model into a multi-firm setting by replacing the Hotelling line with a general standard simplex in high-dimension spaces. The multi-firm setting allows for analysis of various regimes of access charge regulation that are impossible under the rather simple duopoly structure. As is to be detailed below (see Section 4), while in the duopoly setting there exist only two access charges, applied by the two networks with each other, in the three or more firm setting the possible scenarios of interconnection charges are far richer. In the former, we have either symmetric access prices when the two networks charge each other the same interconnection price, or asymmetric access prices when they do not necessarily charge the same price. In the latter, things become far more complicated. For instance, the same firm may apply different access prices to different networks (price discrimination); any pair of networks may charge each other the same access price (reciprocal access charge) or different prices (non-reciprocal); and so on. As a matter of fact, the very terminology of “(a)symmetric access prices” originally coined for the duopoly set-up is not readily applicable in the multi-network setting and needs to be carefully (re)-defined, should one intend to continue to use it.

That the multi-network setting bears importance to reality can hardly be overstated. Following the precedent of the US and England, many other countries have also, more or less, liberalized their own telecommunications markets in the last decade (Lie 2002). Compared with the fixed line telecommunication, the cellular phone industry is much more profitable due to its relatively lower facility cost, and consequently invites more competitors. No doubt, the structure

³ This model has since been successfully applied to analyse a large range of issues such as asymmetric competition (Carter and Wright 2003; Peitz 2005), price discrimination (Laffont, Rey and Tirole 1998b; Gans and King 2001; Hoernig 2007) and heterogeneous demands (Dessein, 2004).

of this highly competitive market varies, sometimes remarkably, across countries. But in an overwhelmingly majority of countries there are more than two firms competing in the market. For instance, by the end of year 2000, in more than half of the OECD countries there are four operators or more in the market, and in only three or four member countries is duopoly found (Paltridge 2000). There have emerged several empirical studies demonstrating that having more firms operating in the market in the OECD economies helps promoting competition and improving the consumer surplus (Langdale 1982; Garfinkel 1993; Dabler, Parker and Saal 2002).

In this paper, we develop a theoretical model, which may be seen as a *natural* extension of the Hotelling duopoly model of Laffont et al (1998a) and Armstrong (1998), to study the multi-network competition and access charge regulation. In the Hotelling duopoly setup the competition occurs between horizontally differentiated goods and services provided by two firms, which are located at the two endpoints of the Hotelling line. The consumers are uniformly distributed along the line. The location of each consumer stands for his ideal preference for the service he would like to buy. The distance between a firm and a consumer denotes the disutility that is incurred if the consumer buys the service from the firm. Such disutility is often interchangeably referred to as the transportation cost in the Hotelling telecommunications model, characterizing the degree of substitutability of two firms' services. We replace the Hotelling line with a (standard) simplex in three-dimension space, preserving the parameter of transportation cost to measure the mismatch cost of the consumer's taste against the service he may receive from the network (our analysis of the access regulation of three firms can be readily further extended to the general multiple firm case, at the cost of cumbersomeness in notation and formulation; our exposition below is therefore largely, though not always, based on the three-firm case). Under given access charges, the firms compete with each other in rentals and per-minute prices, resulting in equilibrium market shares that are essentially determined by the net surplus of the consumers subscribing to each firm. The consumer's net surplus is inclusive of the disutility resulting from mismatch between the services of the networks and the consumer's taste, and hence relates to the latter's location in the simplex. As such, it is the location of the consumers who are indifferent as regards which network to subscribe to that determines the market shares. The societal transportation cost, defined as the sum of all the disutilities incurred by all the consumers, constitutes a loss to the social welfare. We then analyze the efficiency implications of various forms of access charge regulation. To repeat, there are various regimes of

access charge regulation under the multi-firm setting that are absent under the simple duopoly structure. Under all the possible regulation regimes, the most interesting case may be that in which all the access charges are set around a local neighborhood of the cost basis (if not precisely at the cost basis). Our analysis reveals that the efficiency effect of a marginal adjustment of the cost-based access charge largely hinges upon whether and how the equilibrium market shares may be affected thereof. This observation is of great relevance to regulation of a market in which some incumbent firms have been well established but face competition threat from newly emerging entrants. For obvious reasons, whether the new challengers can be viable in taking a firm hold of the market from the incumbents – and what and how the regulation policy may help the entrants to do so – bears importance to promotion of competition and hence consumers' well-being, the very reason motivating all regulation.

As far as we know, very few theoretical studies beyond the Hotelling duopoly setting have been conducted in the literature of competition and regulation of inter-connected networks. The earlier analysis by Salop (1979) of a circular city model of multiple firms suffers from the stringent assumption that competition occurs only between spatially neighboring firms located in a circle. Quite recently, Calzada and Valletti (2008) contribute an interesting analysis of effects of *negotiated* access price between (incumbent) firms, especially on entry deterrence. Not only focusing on different issues, their approach also considerably differs from ours. They adopt a logit demand model in which the preferences of the customers of each firm are assumed to be independently and randomly drawn from an identical double exponential distribution. This assumption enables them to formulate the equilibrium market shares in terms of the networks' call prices and subscription fees (rentals). This approach, however, does not capture the idiosyncratic tastes of the consumers – though it does accommodate the firm-specific idiosyncrasy of the tastes (by an i.i.d. random variable with a zero mean) – and therefore cannot incorporate the social cost that results from the mismatch between the individual consumer's idiosyncratic tastes and the firm's differentiated services. Furthermore, due to the *identical* distribution of the random variable across networks, an assumption that is intrinsically required by the multinomial logit model (see, e.g. Luce and Suppes 1965, p. 338; Anderson, de Palma and Thisse 1992, pp.39-40), equal market shares of all the (incumbent) networks are always obtained in equilibrium. It is worth noting that our simplex-setting is a more *natural* extension of the conventional Hotelling line, in the sense that the disutility resulting from such mismatch between

the consumer's taste and the firm's service, the other things being given, remains crucial in dictating how the market is divided among the networks. Another recent study, Jeon and Hurkens (forthcoming), also generalizes the duopoly model to a multi-firm setting. But their approach is very different from ours too. Rather than deriving the equilibrium market shares, a crucial step and perhaps analytically the most difficult problem, in analysis of multi-network competition and access regulation, they impose some strong assumptions on the so-called market share function (Properties one to five in their model set-up) that make analysis much simpler than what would be required under realistically appealing assumptions. Still, the imposed properties, especially the fifth property, on their market share function only allow for symmetric equilibrium (equal market shares among networks). This restriction on symmetry makes it considerably easier for their exercise to be carried out, but at the cost of abstracting from many interesting issues, for instance the most likely asymmetric market shares between established firms and entrants (see Subsection 4.2 below).

Our "consumer-simplex" approach enables us to obtain several novel results, among which the following may be most worth highlighting. First, we find that when the substitutability of the networks are not too high, or the access charges are close to the marginal cost C_0 , there exists one equilibrium, and the equilibrium is unique under the additional condition that the relative advantages between the networks are not sufficiently significant. Second, under both the uniform and reciprocal access charge regimes (rigorous definitions and category of regulation regimes are found in Section 4), the competition equilibrium under the cost-based access charge maximizes the social welfare, provided that the relative advantages between the networks are not sufficiently significant. The basic reason for efficiency optimization at the cost-based access charge is that the equilibrium market shares remain unchanged with local variations of the access charge around its cost basis. Third, in the particular case that there are two entrants and one incumbent (similar observations obtained for the parallel two-incumbents-and-one-entrant case), the equilibrium market shares no longer remain unaffected by local variations of the access charge around the cost basis. Consequently, when all the access charges are cost-based, a marginal increase in the incumbent firm's non-discriminatory access charge increases the social welfare. Similarly, when all the access charges are cost-based, a marginal increase in the two identical entrant firm's non-discriminatory access charge decreases the social welfare. Interesting as they are, we emphasize that the policy implications of such observations must be drawn with

due caution. An increase in the overall social welfare, through a dominating increase in the incumbent firms' profit at the price of reduction in both the consumer surplus and the entrant firms' profits, is not necessarily beneficial to the society. As is well understood in political economy, both practicability and effectiveness in carrying out the redistributive justice can too often meet serious challenges, especially if and when such redistribution invokes taking money from the long established and powerful companies to compensate for the loss of its competitors, the new entrants, as well as for the "silent majority" of consumers (to borrow Mancur Olson's well known term).

The rest of the paper proceeds as follows. Section 2 sets up the model. Section 3 conducts an analysis of equilibrium that results from price competition between networks, taking the access charges as given. Section 4 studies the efficiency implications of various regimes of access charge regulation and offers some detailed analysis of the particular circumstances under which there are two incumbents (entrants) and one entrant (incumbent). Section 5 concludes.

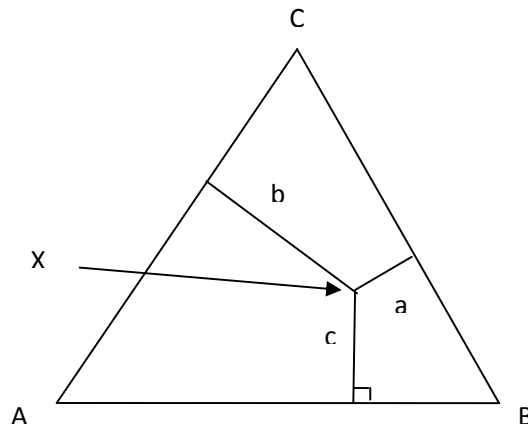
2.The model

2.1. The model set-up

There are three firms in the market, A, B and C, offering services to a continuum of consumers (of which the mass is normalized as one) uniformly distributed on a 2-dimensional simplex I_3 (triangle). The three firms are located at the vertices of the simplex, hence of coordinates $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ respectively, and the services they offer are horizontally differentiated. All the firms practice two-part tariffs, i.e. per-minute price P_i and a fixed fee F_i , $i \in \{A, B, C\}$. That is, each consumer is charged F_i , a fixed cost, plus the calling-minute-dependent cost, $P_i q$ where q represents the minutes of calling. In parallel, each firm incurs two types of costs – the traffic-dependent cost and the connection-dependent cost. The former refers to per-minute cost of originating or terminating a call, denoted as C_0 . That is, if a call originates and terminates on the same network the cost incurred by the network is $2C_0$, and if a call originates from one network and terminates on another, each network incurs C_0 cost.⁴ Let f stand for the connection cost, the cost of connecting to one customer.

⁴ Technically, the network's cost of initializing a call and that of receiving a call are not exactly the same. We make the symmetric cost-sharing assumption, not far from the reality, to facilitate analysis.

As is done in analysis of duopoly structure of telecommunications presented in a Hotelling set-up (Laffont et al 1998a), to study partitioning of the market among the three firms, a notion of “distance” between the consumer and the firm needs to be introduced. For any consumer located at point $X(x_1, x_2, x_3)$, where $x_1 + x_2 + x_3 = 1$ and $x_1, x_2, x_3 \geq 0$, draw perpendicular lines to all the three edges (refer to Figure 1). The perpendicular distances are respectively denoted as a , b , and c . It can be readily shown that across all the consumers $a + b + c$ equals a constant, $\sqrt{6}/2$. For instance, for a consumer located at $(1,0,0)$, the same location of firm A, $a = \sqrt{6}/2$ and $b = c = 0$, while for any consumer located along the edge BC ($x_1 = 0$), $a = 0$ and $b + c = \sqrt{6}/2$. As such, we may take the value of “ a ” as a measure of how close the consumer’s taste is to the service provided by firm A. Note for any two consumers located on a line parallel to edge BC, the corresponding values of “ a ” are the same. But two consumers are located on a line parallel to edge BC if and only if their first coordinates (x_1) are of the same value. We may therefore take the value of x_1 , ranging from zero to one, as a proxy for the closeness of the consumer located at (x_1, x_2, x_3) to firm A, located at $(1,0,0)$ of course.⁵ This may be interpreted as a proxy of the degree of consumer loyalty towards firm A. Alternatively viewed, the distance $(1 - x_1)$ represents the difference between the consumer’s preference (denote by his location) and the service provided by firm A.



⁵ As a matter of fact, the first coordinate of the consumer is neatly related to his perpendicular distance to edge BC in accordance with $x_1 = \frac{\sqrt{6}}{3}a$, or $\frac{\sqrt{6}}{2}(1 - x_1) = \frac{\sqrt{6}}{2} - a$.

Figure 1. Loyalty distance of a consumer to a firm.

The disutility of the consumer located at (x_1, x_2, x_3) from subscribing to firm A is $t * (1 - x_i)$, where t is the cost per unit of distance. The value of t also characterizes the degree of substitutability of services provided by the firms. The larger the value of t , the less substitutable the firms' services. On the other hand, the benefit (in utility terms) of the consumer located at (x_1, x_2, x_3) in subscribing to a network, say network A , is, $u(q) + \theta_i + v_0$. Here, $u(q)$ is the utility derived from calling on the phone for q minutes.⁶ θ_i represents the benefits of subscribing to firm i , and v_0 the fixed surplus of being connected to any network (compared to subscribing to none of them). The value of v_0 is assumed to be quite large to ensure each consumer be connected to a network.

It is, as usual in the literature, assumed that the utility derived from subscribing to a network (network i) and the utility derived from calling are both convertible to monetary units. Thus, the (maximized) net surplus of the consumer who is located at point $X(x_1, x_2, x_3)$ and subscribes to firm i equals,

$$\max_q \{u(q) + \theta_i + v_0 - [P_i q + F_i + t(1 - x_i)]\} \quad (2.1)$$

For the sake of notational convenience below, we introduce

$$w_i(X) \equiv \max_q \{u(q) + \theta_i + v_0 - P_i q - F_i\} \quad (2.2)$$

$$v(P_i) \equiv \max_q [u(q) - P_i q] \quad (2.3)$$

3. Competition and Equilibrium

We study the equilibrium in this section when the access charges between firms are fixed, say exogenously imposed by regulators, or agreed on by the three firms. Let τ_{ij} denote the access charge per minute that firm i receives from firm j when terminating a call originated from firm

⁶ For simplicity we assume consumers do not get utility or disutility from receiving calls, an assumption that can be partially justified on the ground that, statistically, any consumer talks to mobile-mates as long as s/he is talked to on the phone and therefore the form of "benefit-sharing" between the caller and receiver does not seem to be distorting.

j. Like in conventional analysis of the duopoly market structure (e.g. Laffont et al. 1998a), we deal with a two-stage game, but with three leaders rather than two. Furthermore, in each stage the game is played out in a more complicated manner than that under a duopoly scenario. In the first stage, taking the access charges as given, the three firms simultaneously choose their own price strategies (P_i, F_i) , $i = A, B, C$, to maximize profits, hence essentially playing a Bertrand game. In the second stage, having observed these prices, the consumer chooses which firm to subscribe to and how many minutes to call. The consumers' reactions, in terms of choice of the network to subscribe to and decision on the time of calling, are taken into account when the firm, acting as the leaders of the game, choose their price strategies. The standard backward reasoning leads to the sub-game perfect Nash equilibrium.

In the second stage of the game, as is already indicated in the above, the consumer's choice involves two aspects: which network to subscribe to and how much telephone-calling to "consume" (in terms of minutes of calling). That is, for a consumer located at point $X(x_1, x_2, x_3)$ in the standard simplex, the choice is made to maximize his net consumer surplus, as is formulated in equations (2.1) and (2.2) introduced in the preceding section. For some consumers with certain tastes, it doesn't make a difference whether to subscribe to one network or another. Note the consumer's taste is represented by his location. We refer to the consumers indifferent to networks i and j as marginal agents (between these two networks), whose location X is evidently such that $w_i(X) - t(1 - x_i) = w_j(X) - t(1 - x_j)$. The agent who is indifferent to all the three networks is referred to as the indifferent agent of the market. We introduce,

Lemma 1. Denoting the coordinate of the indifferent agent as $X(x_1, x_2, x_3)$ where $x_1, x_2, x_3 \geq 0$ and $x_1 + x_2 + x_3 = 1$, we have,

$$x_i = \frac{1}{3} + \frac{1}{3t} [\sum_{j=1}^3 w_j(X) - 3w_i(X)] \quad (3.1)$$

wherein $w_j(X)$, $j \in \{1,2,3\}$ is defined in equation (2.2).

Proof. For the "indifferent agent", participating in any network brings about the same utility, i.e., $w_i - t * (1 - x_i)$ equals the same value where $w_i = \max_q \{ (a - \frac{b}{2}q)q \} + \theta_i + v_0 - pq -$

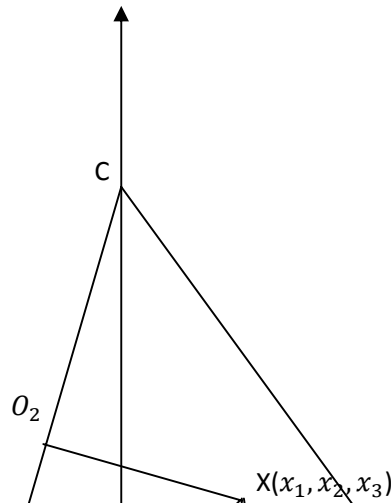
F_i , $i=1, 2, 3$. Thus, $\forall k \in \{1,2,3\}$, $w_k - t(1 - x_k) = \frac{1}{3} \sum_{i=1}^3 [w_i - t(1 - x_i)] = \frac{1}{3} \sum_{i=1}^3 w_i - \frac{2}{3} t$,
hence, $x_k = \frac{1}{3} + \frac{1}{3t} [\sum_{i=1}^3 w_i - 3w_k]$. **QED**

Thus, market partitioning among the three firms naturally follows, dictated by the location of the indifferent consumer,

Lemma 2. Given the non-trivial location of the marginal agent (x_1, x_2, x_3) in the simplex I_3 , where $x_1, x_2, x_3 < 1$, the market shares of the three networks, signified as S_j ($j \in \{A, B, C\}$), are,⁷

$$\begin{cases} S_A = (x_2^2 + x_3^2 + 4x_2x_3)/2 \\ S_B = (x_1^2 + x_3^2 + 4x_1x_3)/2 \\ S_C = (x_1^2 + x_2^2 + 4x_1x_2)/2 \end{cases} \quad (3.2)$$

Proof. Our argument is based on Figure 2.



⁷ Two remarks appear in order here. First, the formula holds only for interior solutions of the market shares. For corner market equilibrium in which one network, or two, are excluded, the scenario, in effect, degenerates into duopoly, or monopoly. Second, a generalized and more complicated formula of equation (3.2) can be made of the equilibrium market shares for *any* number of networks. For instance, in the case of four networks, the market share of the network located at one of the four vertices of the simplex $(1, 0, 0, 0)$, expressed in terms of the indifferent agent's coordinates $X(x_1, x_2, x_3, x_4)$, equals $\frac{1}{3} \sum_{i \neq 1} x_i^3 + \frac{3}{2} \sum_{i,j \in \{2,3,4\}, i \neq j} x_i^2 x_j + 6x_2 x_3 x_4$. Our analysis below is, however, basically confined to the three-firm setting for expositional convenience.

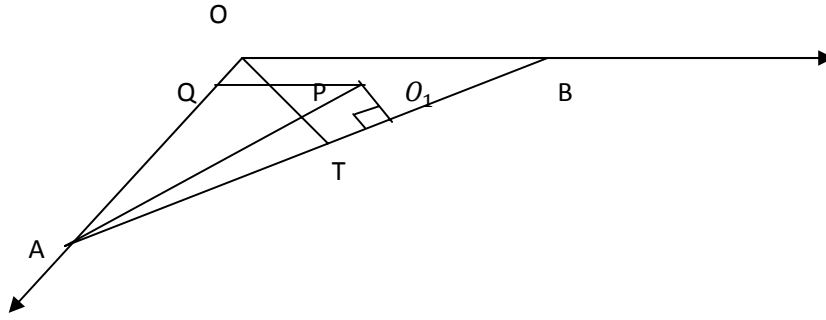


Figure 2. From the “marginal consumer” to market shares: the three-firm case.

Construct a perpendicular to line AB through point $X(x_1, x_2, x_3)$, where at point A the coordinates are $(1, 0, 0)$ and at B $(0, 1, 0)$, and label the point of intersection as O_1 . Also draw a perpendicular XP to the plane OAB, where at point P the coordinates are $(x_1, x_2, 0)$. It can then be shown that $|O_1A|^2 = |PA|^2 - |PO_1|^2 = |PQ|^2 + |QA|^2 - |PO_1|^2$, where point Q is of coordinates $(x_1, 0, 0)$. But $|PQ| = x_2$, $|QA| = 1 - x_1$ and $|PO_1| = \frac{|XP|}{|OC|} \times |OT| = \frac{x_3}{\sqrt{2}}$. Hence,

$$|O_1A| = \sqrt{(1 - x_1)^2 + x_2^2 - \left(\frac{1}{2}\right) [(1 - x_1 - x_2)^2]} = \frac{1 - x_1 + x_2}{\sqrt{2}}. \text{ Note } |XO_1| = \frac{x_3}{1} \cdot \sqrt{\frac{3}{2}}. \text{ Thus the}$$

area of triangle XO_1A , $S_{XO_1A} = \frac{1}{2} (x_3 \sqrt{\frac{3}{2}}) \frac{1 - x_1 + x_2}{\sqrt{2}}$. Similarly, one may obtain point O_2 on the line

AC where $C=(0, 0, 1)$ by constructing a perpendicular to line AC through point O and show that

$$S_{XO_2A} = \frac{1}{2} (x_2 \sqrt{\frac{3}{2}}) \frac{1 - x_1 + x_3}{\sqrt{2}}. \text{ Note the area of the simplex } ABC, S_{ABC} = \frac{1}{2} \cdot \sqrt{\frac{3}{2}} \cdot \sqrt{2}. \text{ As a}$$

consequence, the market share of firm A, $s_A = \frac{S_{XO_1A} + S_{XO_2A}}{S_{ABC}} \times 100\% = \frac{x_2^2 + x_3^2 + 4x_2x_3}{2}$. The

formula for s_B and s_C can be obtained similarly. **QED**

Substituting equation (3.1) into equation (3.2) immediately results in mapping the prices to the market shares,

$$\begin{cases} S_A = \frac{1}{3} + \frac{1}{3t}(2w_A - w_B - w_C) + \frac{1}{6t^2}[(w_A - w_C)(w_A + w_C - 2w_B) + (w_A - w_B)(w_A + w_B - 2w_C)] \\ S_B = \frac{1}{3} + \frac{1}{3t}(2w_B - w_A - w_C) + \frac{1}{6t^2}[(w_B - w_C)(w_B + w_C - 2w_A) + (w_B - w_A)(w_B + w_A - 2w_C)] \\ S_C = \frac{1}{3} + \frac{1}{3t}(2w_C - w_A - w_B) + \frac{1}{6t^2}[(w_C - w_A)(w_C + w_A - 2w_B) + (w_C - w_B)(w_C + w_B - 2w_A)] \end{cases} \quad \text{----- (3.3)}$$

In the first stage of the game, the three firms move simultaneously in choosing strategic prices $\{P_i, F_i\}$, $i = A, B, C$. Consider the choice of firm A. The profit is composed of three parts. The first part is dependent on the quantity of consumption of network A's subscribers and the piece-rate charge on the calls. Since the per-minute price remains the same no matter which network the receiver of the call may subscribe to, the caller does not exert any discrimination between receivers of different networks. Thus, the profit resulting from calls originated from and terminated at network A (on-net call) equals $S_A q_A [S_A (P_A - 2C_0)]$. Note the firm incurs double C_0 for per-minute calling if both the caller and the receiver are subscribing to network A. The profit resulting from calls originated from network A and terminated at network B or C equals $S_A q_A [S_B (P_A - C_0 - \tau_{BA}) + S_C (P_A - C_0 - \tau_{CA})]$. Here firm A incurs, in addition to the traffic cost C_0 caused by its subscriber, the access charge paid to firm B, τ_{BA} , and the access charge paid to firm C, τ_{CA} . The second part of firm A's profit comes from access charge for incoming calls from firm B and C when their subscribers call consumers who subscribe to firm A, equaling $S_A [S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0)]$. Firm A incurs the traffic cost C_0 since its subscribers receive the calls. The third part of the profit comes from fixed fees net of the connection costs, equaling $S_A (F_A - f)$. The first two parts of the profit function refer to what can be realized in *deepening* the market (affecting the intensive usage of the network by each individual consumer), while the third part refers to what can be realized in *expanding* the market share (affecting the extensive margin, namely the number of subscribers of the network). Firm A's profit thus equals (similar formulation applies to firms B and C, omitted here),

$$\pi_A = S_A q_A [S_A (P_A - 2C_0) + S_B (P_A - C_0 - \tau_{BA}) + S_C (P_A - C_0 - \tau_{CA})] + S_A [S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0)] + S_A (F_A - f) \quad (3.4)$$

In the above formula, q_i only depends on the per-minute price decided by firm i , $i \in \{A, B, C\}$. The market share of firm A, S_A , is determined jointly by the three firms' price strategies, $\{(P_i, F_i), i = A, B, C\}$. Taking P_B, F_B, P_C and F_C as given, firm A optimizes on its prices (P_A, F_A) . Leaving the algebraic details to Appendix A, we obtain (the equilibrium solution is still denoted as (P_A, F_A) for the sake of notational simplicity),

$$P_A = S_A(2C_0) + S_B(\tau_{BA} + C_0) + S_C(\tau_{CA} + C_0) \quad (3.5A)$$

$$F_A = f + \frac{tS_A}{x_2+x_3} + \frac{x_3}{x_2+x_3} S_A q_B (\tau_{AB} - C_0) + \frac{x_2}{x_2+x_3} S_A q_C (\tau_{AC} - C_0) - [S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0)] + S_A q_A \left[\left(\frac{x_3}{x_2+x_3} \right) (C_0 - \tau_{BA}) + \left(\frac{x_2}{x_2+x_3} \right) (C_0 - \tau_{CA}) \right] \quad (3.5B)$$

As a consequence, the equilibrium profit of firm A is

$$\pi_A = S_A [S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0)] + S_A (F_A - f) \quad (3.5C)$$

Note that the perceived marginal cost pricing still holds on the per-minute price P_A , as reflected in (3.5A). This is consistent with the marginal cost pricing rule in the Hotelling model of two networks (Laffont, et al 1998a). A firm charges the per-minute price according to the anticipated marginal cost, instead of the marginal cost of the whole industry. Under the balanced calling pattern, the percentage of callings originating from, say firm A, and terminating on any network, equals the latter's market share. Note also, for per-minute on-net call the network incurs a cost of $2C_0$. For per-minute outgoing call the network incurs a cost of C_0 plus an access charge paid to the network terminating the call. Therefore, the perceived marginal costs per one-minute calling originating from network A and terminating on networks A, B and C, respectively, are $S_A(2C_0)$, $S_B(\tau_{BA} + C_0)$ and $S_C(\tau_{CA} + C_0)$.

As to the fixed fee, F_A , the economic intuition is also not difficult to grasp, despite its seemingly complicated expression in Eq. (3.5B). A change in F_A affects the profit largely by causing a (marginal) change of the market-share landscape. Recall that the profit function Eq. (3.5) is composed of three parts, respectively resulting from outgoing calls, incoming calls and fixed fees. Correspondingly, a marginal increase of F_A leads to changes in the three parts. The rendered change in the profit reaped from the outgoing calls, in view of the perceived marginal

cost pricing (3.5A), equals $\frac{\partial S_A}{dF_A} q_A [S_A(P_A - 2C_0) + S_B(P_A - C_0 - \tau_{BA}) + S_C(P_A - C_0 - \tau_{CA})] + S_A q_A \left[\frac{\partial S_A}{dF_A} (P_A - 2C_0) + \frac{\partial S_B}{dF_A} (P_A - C_0 - \tau_{BA}) + \frac{\partial S_C}{dF_A} (P_A - C_0 - \tau_{CA}) \right] = S_A q_A \left[\frac{\partial S_B}{dF_A} (C_0 - \tau_{BA}) + \frac{\partial S_C}{dF_A} (C_0 - \tau_{CA}) \right]$. The rendered change in the profit reaped from the incoming calls equals $\frac{\partial S_A}{dF_A} [S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0)] + S_A \left[\frac{\partial S_B}{dF_A} q_B (\tau_{AB} - C_0) + \frac{\partial S_C}{dF_A} q_C (\tau_{AC} - C_0) \right]$. The rendered change in the profit reaped from the fixed fees is $\frac{\partial S_A}{dF_A} (F_A - f) + S_A$. Gathering all the changes and noting that $\frac{\partial S_A}{dF_A} = \frac{x_1 - 1}{t}$, $\frac{\partial S_B}{dF_A} = \frac{x_3}{t}$ and $\frac{\partial S_C}{dF_A} = \frac{x_2}{t}$, we obtain (3.5B) after some algebraic manipulation (detailed computation is found in Appendix A).

For given values of (P_B, F_B, P_C, F_C) , the market shares, as well as the location of the consumer who is indifferent to any network, denoted as $X(x_1, x_2, x_3)$, are jointly determined by (P_A, F_A) . Equations (3.5A) and (3.5B) do not provide an explicit solution of Firm A's profit optimization problem. Nevertheless, based on these equations, the existence and uniqueness can be guaranteed under plausible conditions. We posit,

Proposition 1. When t is sufficiently large, i.e. the substitutability of two networks are not too high, or the access charges are close to the marginal cost C_0 , there exists one equilibrium; furthermore, if the relative advantages between the networks are not sufficiently significant, the equilibrium is unique, as is characterized by (3.5).

Proof: Found in Appendix A.

To come to grips with the efficiency analysis of the Nash equilibrium, technically, we need to find the total "transportation cost" at equilibrium of all the consumers.

Lemma 3. The total transportation cost of all the consumers when the indifferent consumer is located at (x_1, x_2, x_3) equals $\frac{t}{2} (1 - 6x_1 x_2 x_3)$.

Proof: Making use of Figure 3, we now calculate the total transportation costs of all consumers using network A. All the users of network A fall within the polygon ADXG, where the indifferent consumer is located at point $X(x_1, x_2, x_3)$. To facilitate computation, we first hypothetically assume all the consumers falling within the triangle AEF are network A users and

find the total transportation costs of all of them, and then subtract the costs of those located within triangles GXF and DXE. Note that, as seen from the proof of Lemma 2, the coordinates of points D and G respectively are $(\frac{1-x_2+x_1}{2}, \frac{1-x_1+x_2}{2}, 0)$ and $(\frac{1-x_3+x_1}{2}, 0, \frac{1-x_1+x_3}{2})$. For each consumer whose first coordinate is w , the distance to firm A is $(1-w)\frac{\sqrt{6}}{2}$, hence the transportation cost equaling $(1-w)\frac{\sqrt{6}}{2}t$. Simple reasoning reveals that the population density over the triangle ABC is $\frac{2}{\sqrt{3}}$. But the differential area when w changes infinitesimally (refer to Figure 3) is $(1-w)\sqrt{2}dw$. Thus, the total costs of the consumers falling with triangle AEF, if they all subscribe to network A, equal $\int_{x_1}^1 [(1-w)\frac{\sqrt{6}}{2}t] \cdot \frac{2}{\sqrt{3}} \cdot (1-w)\sqrt{2}dw = \frac{2}{3}t(1-x_1)^3$. Similarly, the transportation costs of consumers within triangles GXF, if subscribing to network A, equal $\int_{x_1}^{\frac{1-x_3+x_1}{2}} [(1-w)\frac{\sqrt{6}}{2}t] \cdot \frac{2}{\sqrt{3}} \cdot [(\frac{1-x_3+x_1}{2}-w)\frac{\sqrt{6}}{2} \cdot \frac{4}{\sqrt{3}}] dw = \int_{x_1}^{x_1+\frac{x_2}{2}} 4(1-w)t(x_1+\frac{x_2}{2}-w)dw = \frac{x_2^2}{2}(1-x_1-\frac{x_2}{6})t$. By symmetry, the transportation costs of consumers within triangle DXE, if subscribing to network A, equal $\frac{x_3^2}{2}(1-x_1-\frac{x_3}{6})t$. Hence, the total transportation costs of all consumers using network A is, $\frac{2}{3}t(1-x_1)^3 - \frac{x_2^2}{2}(1-x_1-\frac{x_2}{6})t - \frac{x_3^2}{2}(1-x_1-\frac{x_3}{6})t = (x_2+x_3) \cdot \frac{(x_2+x_3)^2+3x_2x_3}{4} \cdot t$.

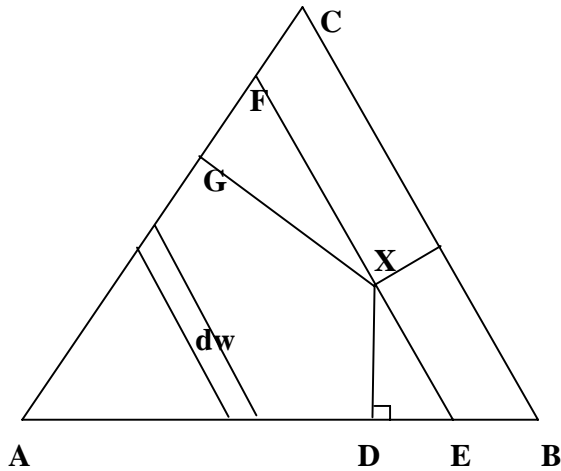


Figure 3. Transportation costs.

By similar reasoning, we find the total transportation costs of all the consumers using networks B and C respectively are $(x_1 + x_3) \cdot \frac{(x_1+x_3)^2+3x_1x_3}{4} \cdot t$ and $(x_1 + x_2) \cdot \frac{(x_1+x_2)^2+3x_1x_2}{4} \cdot t$. Summing up all the transportation costs of consumers subscribing to firms A, B and C yields $\frac{1-6x_1x_2x_3}{2} \cdot t$. **QED**

For some, but not all, of the analyses to be conducted below, it suffices to note that the societal transportation cost is a continuous function of the coordinates of the indifferent agent (x_1, x_2, x_3) . It is also interesting to note that the above formula for explicitly computing the societal transportation costs is of interest in itself.⁸

4. Access charges and efficiency

We now analyze efficiency implications of the interconnection charges. Different from the duopoly setting in which there exist only two access charges, applied by the two networks with each other, in the three or more firm setting the possible scenarios of the interconnection charges are far richer. In the former, we have either symmetric access prices when the two networks charge each other the same interconnection price, or asymmetric access prices when they do not necessarily charge the same price. As is shown in more detail shortly, in the three-firm setting we face far more nuanced interconnection prices. As a matter of fact, the very terminology of “(a)symmetric access prices” is not readily applicable in the multi-network setting and needs to be carefully (re)-defined, should one intend to continue to use it.

In principle when there exist n networks, there are at most $n(n - 1)$ access price instruments. In our three-network setting in particular, there exist six possibly different access prices, i.e. $\tau_{AB}, \tau_{AC}, \tau_{BA}, \tau_{BC}, \tau_{CA}$ and τ_{CB} , where τ_{ij} is the access charge network i receives from network j . Depending on the rules we may impose on the relationships between the values of the six instruments, be the rules themselves determined by the regulator, inter-firm agreement, or

⁸ As a matter of fact, the formula can be generalized to the setting with any number of networks, a generalization omitted here.

some special acts existing in certain economies, we have different regimes of access prices. Of particular interest, however, may be the following. In the simplest case, all networks are required to uniformly apply the same access price to any other firm. The six price variables become one, i.e. $\tau_{ij} = \tau$ for any $i \neq j$, and τ will hereafter stand for the uniform access charge unless stated otherwise. It is also possible that any two networks charge each other the same access price, for instance, as required by certain policies or simply by agreement between firms, i.e. $\tau_{ij} = \tau_{ji}$, for any $i \neq j$. That naturally results in a reciprocal access charge regime, in which only three different prices, τ_{AB} , τ_{BC} and τ_{CA} , are charged between the networks. Another interesting setting in which there are also three possibly different access charges being practiced is that each firm exercises a non-discriminatory price to the other two networks. Of particular interest to policy analysis is the situation in which there exists one or more incumbent networks, as well as some new entrants. For example, there may be the case that one firm is well established in the telecommunications industry but two new entrants, of a same size but much smaller than the established firm, enter the market. It seems appealing in this case to allow the two entrants to charge the same price for interconnection access from the “big brother” network and the latter applies the same price (non-discrimination price) to the former. A *de facto* coalition between the two entrants is formed. Clearly this access price regime differs from each of those mentioned in the above. As a matter of regulation, this regime invokes more discretion in policy formulation. For the sake of convenience, we shall refer to this situation below as a “discretionary regime (in access charge)”, in the hope that, given the contexts, such references will not cause misunderstanding but facilitate the exposition. It goes without saying that there are more alternatives of the interconnection price regimes. For reasons to be seen below, we shall nonetheless concentrate our attention only upon the aforementioned scenarios, which for the sake of illustration are summarized in Table 1.

Table 1. A non-exclusive category of interconnection access price regimes.

Uniform access charge	Reciprocal access charge	Non-Discriminatory access charge	Discretionary access charge
$\tau_{ij} = \tau, \forall i \neq j$	$\tau_{ij} = \tau_{ji}, \forall i \neq j$	$\tau_{ij} = \tau_i$, regardless of j	$\theta_A \neq \theta_B = \theta_C$, $\tau_{AB} = \tau_{AC}, \tau_{BA} = \tau_{BC}$

We shall first consider in Subsection 4.1 the relatively simpler access charge regimes: the uniform access price and the reciprocal access prices. We then consider in Subsection 4.2 the scenario in which there are one incumbent and two identical entrants, as well as the parallel case wherein there are two incumbents and one entrant. Not surprisingly, some existing results in the literature for models with two competing firms (Laffont et al 1998a, Carter and Wright 2003, Peitz 2005) do not readily extend to a multi-firm setting.

4.1 The uniform access charge and reciprocal access charges

Under the uniform access charge regime, the same price is used by each firm for interconnection access from any other network. The cost-based access charge stands as the most economically appealing, as in the two-network setting (e.g. Carter and Wright 2003, Peitz 2005). Of our primary concern is the efficiency implication of such a pricing rule. For that purpose, we need to first understand whether the equilibrium market shares are affected by marginal changes of the cost-based access charge.

Lemma 4. The equilibrium market shares remain unchanged when the uniform access charge marginally changes around the cost basis.

Proof: Found in Appendix B.

The economic intuition of the above lemma is not difficult to grasp. A marginal increase of the uniform access charge from the cost basis results in a slight increase in each firm's per-minute price, as shown in equation (3.5A). The larger the firm's equilibrium market share, the smaller the increase in its per-minute price, since the latter equals the perceived marginal cost of outgoing calls. The decrease in the rental (F_i) is equally but inversely related in proportion to the market share. Thus, the increase in the per-minute price and the decrease in the rental offset with each other, leaving each consumer's utility unchanged, whichever network he subscribes to.

We now turn to the resulting social welfare, which is defined in a standard manner as the sum of the consumer surplus and the producers' profits.

Proposition 2. Under the uniform access charge regime, the cost-based access charge (locally) maximizes the social welfare, provided that the relative advantages between the networks are not sufficiently significant.

Proof: Found in Appendix B.

To intuitively understand why the cost-based uniform access price maximizes the social welfare as it does, one may consider the simplest case in which the three networks are of precisely the same appeal to the consumers. The markets are then equally divided among the trio. As always, the consumers' loss in terms of monthly rentals (F_i) is the producers' gain, and one firm's loss in terms of access charge is another firm's gain. Thus, in the light of perceived marginal cost pricing, in assessing the societal surplus only the profit of each firm that is reaped from incoming calls, and the indirect utility of each consumer of the firm derived from calling matter. We then only need to focus on one network, say network A, and consider $G(\tau) \equiv v(P_A) + [S_B q_B(\tau - C_0) + S_C q_C(\tau - C_0)] = v(P_A) + [q_B(\tau - C_0) + q_C(\tau - C_0)]/3$. The consumer's indirect utility $v(P_A)$ decreases in the access charge (of which the burden is of course ultimately born by the consumers) but the firm's gain from incoming calls (refer to the above formula for $G(\tau)$) increases with the charge. By the same token, the consumers' consumption of the network service (q_B and q_C) decreases in the access charge τ . At the cost-based access charge, the consumer's loss equals the firm's gain, resulting in $\frac{\partial G(\tau)}{\partial \tau} \Big|_{\tau=C_0} = 0$. $G(\tau)$ is locally maximized since $\frac{\partial^2 \partial G(\tau)}{\partial \tau^2} \Big|_{\tau=C_0} = \frac{1}{3} \left[\frac{\partial q_B}{\partial \tau} \Big|_{\tau=C_0} + \frac{\partial q_C}{\partial \tau} \Big|_{\tau=C_0} \right] < 0$. This is also the case for the social welfare. By continuity, this also applies to the situation when the three networks' services do not significantly differ from one another.

Under the reciprocal access price regime, there are three possibly different prices. Around the cost basis, a marginal change in the reciprocal charge between any pair of networks does not affect the equilibrium market shares. Furthermore, fixing up two cost-based reciprocal charges and allowing for local changes in the other charge, the latter, under plausible conditions, maximizes the social welfare at the cost basis. These observations are formally stated in Lemma 5 and Proposition 3 as follows, of which not just the rigorous proofs but also the economic intuition are similar to that for Lemma 4 and Proposition 2.

Lemma 5. When all the access charges are cost-based, a marginal change in the reciprocal access charge between any two networks does not affect the equilibrium market shares.

Proof: Found in Appendix B.

Proposition 3. When all the access charges but for that between two networks are fixed at the cost basis, social welfare is maximized when the access charge between the two networks is also cost based, provided that the relative advantages between the networks are not sufficiently significant.

Proof: Found in Appendix B.

4.2 Some particular access charge regimes when two networks are identical

A particular situation, of significant interest to both policy design and theoretical analysis, is that in which there are one well established incumbent firm and two new entrants, or there are two existing firms that have long been in operation in the market and one entrant. Diffusion of a new technology often displays a logistic curve, as is well historically documented (see e.g. Griliches' (1957) seminal study of the adoption of the hybrid corn technology). One most relevant aspect of the increasingly wide use of mobile phones to access charge regulation is that as the market expands there naturally emerge newcomers that intend to enter and compete with the existing firms for the market. We consider two particular regimes of access charges that appear to be suitable to the situation in which there are two identical firms (be they entrants or incumbent firms) and one different firm: the "discretionary access charge" and the non-discrimination access charges (both have been briefly described earlier). There are three independent access prices under the discretionary regime and two independent access prices under the non-discriminatory access charge regime. Both for the sake of analytical neatness and relative importance of the "1-to-2" case, we shall conduct somewhat detailed analysis of the one-incumbent-and-two-entrants ("1-to-2") case, leaving the parallel case of two-incumbents-and-one-entrant (the "2-to-1" case) only briefly addressed.⁹

⁹ One important difference between the "1-to-2" and "2-to-1" cases is that in the latter there always exists somewhat keen competition between two equal-sized incumbent firms while in the former the market structure could be nearly monopoly when the two entrants turn out to be weak/small enough in competing with the single incumbent firm. Thus, whether the entrants are viable in competing with the established in the former setting implies a choice between oligopoly and nearly-monopoly, but it is not the case in the latter.

The discretionary access charges

Given the identity of the two firms, under this regime, we simplify the access charges into three types. τ_1 is the access charge that the incumbent, firm A, charges each of the two identical entrants, firms B and C, for terminating calls on its network. τ_2 is the access charge that each entrant charges the incumbent for terminating the calls on its network. τ_3 is the access charge between the two identical entrants. That is, the incumbent exercises a non-discriminatory access charge (τ_1), but the entrants do not necessarily do so (i.e. it is not necessary that $\tau_2 = \tau_3$).

Before coming to terms with the efficiency implications of the access charges, we need to study how equilibrium market shares respond to marginal changes in a neighborhood of any one of the three cost-based access charges. We posit,

Lemma 6. When all the access charges are cost-based,

- (i). A marginal increase in the incumbent firm's access charge increases its equilibrium market share.
- (ii). A marginal increase in the access charge of the two entrants to the incumbent increases the entrants' equilibrium market shares.
- (iii). A marginal change in the access charge between the two entrants has no effect on the equilibrium market shares.

Proof: Found in Appendix C.

The effect of τ_1 (τ_2 , or τ_3) on the equilibrium market shares is not hard to intuitively understand. One only needs to consider the surplus of the indifferent consumer, whose coordinates are denoted as $X(x_1, x_2, x_3)$ and for whom therefore, $w_A(X) = w_B(X) = w_C(X)$. Simple analysis invoking equations (3.5A) and (3.5B) reveals that $\partial[w_A(X) - w_B(X)]/\partial\tau_1|_{\tau_1=\tau_2=\tau_3=C_0} > 0$, $\partial[w_A(X) - w_B(X)]/\partial\tau_2|_{\tau_1=\tau_2=\tau_3=C_0} < 0$, and $\partial[w_A(X) - w_B(X)]/\partial\tau_3|_{\tau_1=\tau_2=\tau_3=C_0} = 0$. By continuity, the agents within a local neighborhood of the indifferent agent, including those who previously subscribe to network B or C, will all go to network A when τ_1 slightly increases from its cost basis while $\tau_2 = \tau_3 = C_0$. Similarly, some agents will switch from network A to B or C when τ_2 slightly increases from its cost basis while $\tau_1 = \tau_3 =$

C_0 , and no change in the customer-firm relationship landscape when τ_3 slightly changes from its cost basis while $\tau_1 = \tau_2 = C_0$.

Equipped with the above Lemma, we are now able to tackle the efficiency implication of marginal changes of each of the three cost-based access charges.

Proposition 4. In an asymmetric market with one incumbent and two entrants, when all the access charges are cost-based the social welfare increases with the incumbent firm's access charge (τ_1), decreases with the each entrant firm's access charge to the incumbent firm (τ_2), but remains unchanged with the access charge between the two entrant firms (τ_3).

Proof: Found in Appendix C.

To economically understand why the social welfare behaves in response to variation in access charges in the manner described above, we need to break down the social welfare and look at the behavior of each of its components. We may first consider the effects of a marginal change in τ_1 around the cost basis (C_0) on the consumer surplus (CS) and the firms' profits π_A (the incumbent's profit) and π_B (each entrant's profit). After some algebra manipulation (omitted, available upon request), we can show that $\left. \frac{\partial CS}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} < 0$, $\left. \frac{\partial \pi_A}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} > 0$, and $\left. \frac{\partial \pi_B}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} < 0$. That is, both the consumer surplus and the two entrants' profits decrease, but the incumbent firm's profit increases. However, the increase in the latter dominates the former two negative effects combined, hence social welfare is enhanced. When τ_2 slightly increases from C_0 while keeping $\tau_1 = \tau_3 = C_0$, the consequence is reversed. That is, both the consumer surplus and the entrants' profits increase, while the incumbent's profit not just decreases, but decreases more quickly than the increase in the consumer surplus and the entrants' profits combined. Hence, social welfare declines. It may be noted that the key element that drives the overall effect of a marginal change in τ_1 (or τ_2) from its cost basis is that the equilibrium market shares change with such access price, as is demonstrated in Lemma 6. By the same token, a local change of τ_3 around its cost basis does not make a difference in any agent's surplus, hence rendering no difference in the equilibrium market shares (as is also shown in Lemma 6).

Moreover, simple computation reveals that it does not cause any change in the profit of any firm. Thus, social welfare remains unchanged in a local change of τ_3 around its cost basis.¹⁰

Similar analysis to that for Lemma 6 and Proposition 4, as well as appropriately modified conclusions, may be made for the “2-to-1” case. To save space, we leave it to interested readers.

Non-discriminatory access charges

Under this regime, each network applies the same access charge to the other two networks. Differing from the “discretionary regime” considered in the above, τ_2 and τ_3 are always of the same value under the non-discriminatory regime. That is, each entrant charges the same price to the incumbent firm as well as to the other entrant. We denote by τ_A the incumbent network A’s access charge to the two entrants and by τ_B each entrant’s access charge to the other firms under the non-discriminatory regime.

Once again, an interesting question that merits a careful examination is: what’s the consequence of a marginal change of one network’s access charge around the cost basis? In particular, does it make a difference in the equilibrium market shares?¹¹ We have,

Lemma 7.

(i).When all the access charges are cost-based, a marginal change in the non-discriminatory access charge of the firm that is different from the other two firms positively affects its equilibrium market share.

(ii).Similarly, when all the access charges are cost-based, a marginal change in the two identical firms’ non-discriminatory access charge positively affects their equilibrium market shares.

Proof: Found in Appendix C.

It is important to point out that the non-discrimination access charges used by the two entrants are assumed to be always the same due to symmetry between them, an assumption that renders the analysis possible.¹²

¹⁰ In fact, the cost based τ_3 maximizes social welfare under the circumstance that both τ_1 and τ_2 are fixed at the cost basis.

¹¹ It is worth noting that the non-discriminatory regime is not analytically solvable in the general setting with three heterogeneous firms. That notwithstanding, it suffices, for our purpose, to obtain Lemma 7, as well as Proposition 5 below, for the special case that two among the three networks are identical.

Lemma 7 may appear at first sight counter-intuitive in that a marginal change in the incumbent's, or the entrants', cost-based access charge alone alters the equilibrium market shares, but when all the access charges change in a uniform manner the market shares remain unaffected as is implied by Lemma 4.¹³ It is explicable, however, as follows. First, different from the uniform access charge regime, an increase in the incumbent's access charge that is received from the entrants, directly translates into an increase in the per-minute price of the latter (refer to equation (3.5A)) but not in that of the former. Despite adjustment in the rentals of both the incumbent and the entrants, the net surplus of any consumer of the incumbent is thus given an edge over that of those subscribing to an entrant, hence tilting the market shares toward the incumbent firm. Secondly, for a marginal change of the two entrants' uniform access charge around the cost basis, essentially the same mechanism, but to a lesser extent, is at work. Uniformly increasing $\tau_{BA}, \tau_{BC}, \tau_{CA}$ and τ_{CB} (all are denoted as τ_B in the proof of Lemma 6 in Appendix C) from their cost basis results in a larger change in P_A than in P_B or P_C . By equation (3.5A), assuming τ_B increase by Δ , the increase in P_A equals $(S_B + S_C)\Delta$, while the changes in P_B and P_C respectively equal $S_B\Delta$, and $S_C\Delta$. Consequently, a marginal change of the entrants' access charge around the cost basis lends an edge to the new entrants' consumers over those subscribing to the incumbent. Consequently, the entrants gain slightly more of the market from the incumbent.

Exploiting Lemma 7, we can now analyze the efficiency implication of marginal changes in the incumbent's (entrants') access charge under the 1-to-2 setting.

Proposition 5.

- (i). When all the access charges are cost-based, a marginal increase in the incumbent firm's non-discriminatory access charge increases the social welfare.
- (ii). Similarly, when all the access charges are cost-based, a marginal increase in the two identical entrant firm's non-discriminatory access charge decreases the social welfare.

¹² Another scenario that we will not analyze in this study is that in which the two identical entrants may charge possibly different access prices.

¹³ It also stands in contrast with a known result in the literature under the two-firm setting that the entrant's access charge, when changing locally around the cost, does not affect the market shares (Peitz 2005). The same observation that the firms' market shares remain unchanged when the access charge changes locally around cost is also implied by an intermediate result in Carter and Wright (2003, p.35).

Proof: Found in Appendix C.

The underlying reason that social welfare varies with marginal changes in the access charges is that the equilibrium market shares vary with marginal changes in the access charges. When the incumbent's access charge increases infinitesimally, the incumbent's profit is increased infinitesimally too. But the entrants' profits and the societal consumer surplus both decrease. However, the former (positive) effect dominates the latter (negative) effect. Consequently, social welfare increases. By contrast, the effect of increasing the entrants' access charge is just opposite.¹⁴

It may be noted that our analysis so far of the non-discriminatory access charge regime, as well as of the discretionary charge regime, is confined to the local neighborhood of cost-based access prices. Despite that cost based access charges are not only economically appealing but rendering formal analysis possible, one may nonetheless wonder what would happen if the access charges are well off the cost basis. Unfortunately, it appears impossible to analytically solve in the general case the social welfare optimization problem, even when two variables of $\{\tau_1, \tau_2, \tau_3\}$, for instance say τ_2 and τ_3 , are fixed at C_0 , and only τ_1 is to be arbitrarily chosen. As is discussed above, locally increasing τ_1 around its cost basis increases the incumbent firm's profit, but decreases both the consumer surplus and the entrants' profits, with the incumbent firm's gain outweighing the consumers' loss and the two entrants' loss combined, and thus resulting in enhancement of social welfare. However, such a statement cannot generally extend when τ_1 is well above C_0 . As shown in Section 3, each firm's profit comes from incoming calls and rentals (formula 3.5C). An increase in the incumbent firm's access charges translates into increase of the entrants' per-minute prices (formula 3.5A) and thereby forces some agents to switch from the entrant firms to the incumbent. The incumbent firm's marginal gain in rentals from an enlarged consumer base (an increased market share) dominates the marginal loss from incoming calls by clients of the entrant networks when τ_1 slightly increases from C_0 . As τ_1 becomes larger and larger, however, the tradeoff is tilted in favor of the marginal cost. In other words, even leaving the incumbent free in choosing its access charge while fixing both τ_2 and τ_3 at C_0 , the firm maximizes its profit at a certain level of τ_1 that is not infinitely large but dictated by the

¹⁴ Parallel observations to the above lemma and proposition may be made for the "2-to-1" case (two incumbents and one entrant).

incumbent's relative advantage over the new entrants. The larger the advantage, the larger the optimal access charge.¹⁵ To illustrate this point, we conduct a numerical simulation in which we assume a quadratic form of utility function of making calls, $u(q) = \left(a - \frac{b}{2}q\right)q$. We set $a = 185$, $b = 0.54$, $t = 60000$, $f = 20000$ and $v_0 = 50000$, and summarize the simulation results in Table 2.¹⁶ The values of parameters chosen here are mainly illustrative.

Table 2. A simulation of the upper bound of flexible access charge τ_1 when $\tau_2 = \tau_3 = C_0$

$\theta_B = \theta_C$	θ_A	The value of τ_1 that maximizes social welfare	The value of τ_1 that maximizes π_A
5000	6000	12	240
	8000	15	236
	11000	20	230
	14000	23	225
	17000	27	219
	20000	29	214
	23000	32	209
	26000	35	204
	30000	36	198
	33000	38	193
	36000	39	189
	40000	41	183
45000	43	176	

As is shown in the table, not surprisingly, the access charge that maximizes the incumbent's profit is higher than that maximizes the social welfare. However, the gap between these two accesses charges decrease as the incumbency advantage increases. This is because the incumbent's market share increases with its incumbency advantage. As the incumbency advantage increases, the incumbent's profit accounts more in the social welfare. As a result, the gap between these two optimal access charges reduces.

Another point, closely related to the above, we would like to make is that an increase in the overall social welfare, through a dominating increase in the incumbent firms' profit at the price of reduction in both the consumer surplus and the entrant firms' profits, is not necessarily

¹⁵ The optimal charge for the firm's profit is not necessarily that which maximizes the social welfare.

¹⁶ The simulation program was written in Matlab version 2007b (available upon request from the authors).

beneficial to the society. As is well understood in political economy, both practicability and effectiveness in carrying out the redistributive justice can too often meet serious challenges, especially if and when such redistribution invokes taking money from the long established powerful companies to compensate for the loss of its competitors, the new entrants. The powerlessness of the consumers as a silent majority a la Mancur Olson only exacerbates the difficulty.

5. Conclusion

In this paper, we develop a framework, naturally extended from the Hotelling duopoly model of Laffont et al (1998a) and Armstrong (1998), to study the multi-network competition and access charge regulation. Replacing the Hotelling line with a simplex over which the consumers are located enables us to formulate the market shares at competitive equilibrium, and thereby exploring the efficiency implications of various regimes of access charge regulation. Our major findings may be summarized as follows. First, when the substitutability of the networks are not too high, or the access charges are close to the marginal cost C_0 , there exists one equilibrium, and the equilibrium is unique under the additional condition that the relative advantages between the networks are not sufficiently significant.

Second, under *both the uniform and reciprocal access charge regimes*, the competition equilibrium under the cost-based access charge maximizes the social welfare, provided that the relative advantages between the networks are not sufficiently significant. The first part of this finding may be understood as generalizing the known result in the Hotelling duopoly setup that cost-based *uniform access charge* maximizes efficiency, while the second part, as regards reciprocal access charges, does not have a counterpart in the duopoly story since there are several admittedly different reciprocal charges under the multi-firm set-up.

Third, in the particular case that there are two entrants and one incumbent (similar observations obtained for the parallel two-incumbents-and-one-entrant case), a marginal increase in the incumbent firm's non-discriminatory access charge increases the social welfare while a marginal increase in the two identical entrant firm's non-discriminatory access charge decreases the social welfare. The reason is that the market share at competition equilibrium is affected by a marginal change in the cost-based access charge. This contrasts with the existing studies of the duopoly structure in which the market share remains unchanged (as discussed already in

Subsection 4.2). It is also in order to emphasize that a certain measure of caution must be exercised when drawing the policy implications from the above observation. For an increase in the overall social welfare, through a dominating increase in the incumbent firms' profit at the price of reduction in both the consumer surplus and the entrant firms' profits, is not necessarily beneficial to the society.

Appendices

Appendix A.

The proof of Proposition 1. We proceed in three steps.

Step 1, derivation of the equilibrium. Consider firm A's profit maximization problem,

$$\pi_A = \max_{P_A, F_A} S_A q_A [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + S_A S_B q_B (\tau_{AB} - C_0) + S_A S_C q_C (\tau_{AC} - C_0) + S_A (F_A - f)$$

The first-order conditions (FOC) are,

$$\begin{aligned} \frac{\partial \pi_A}{\partial P_A} = \frac{\partial S_A}{\partial P_A} \{q_A [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0) + F_A - f\} + S_A \cdot \left\{ \frac{\partial q_A}{\partial P_A} [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + q_A \left[1 + \frac{\partial S_B}{\partial P_A} (C_0 - \tau_{BA}) + \frac{\partial S_C}{\partial P_A} (C_0 - \tau_{CA}) \right] + \frac{\partial S_B}{\partial P_A} q_B (\tau_{AB} - C_0) + \frac{\partial S_C}{\partial P_A} (\tau_{AC} - C_0) \right\} = 0 \end{aligned} \quad (A1)$$

$$\begin{aligned} \frac{\partial \pi_A}{\partial F_A} = \frac{\partial S_A}{\partial F_A} \cdot \{q_A [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0) + F_A - f\} + S_A \left\{ q_A \left[\frac{\partial S_B}{\partial F_A} (C_0 - \tau_{BA}) + \frac{\partial S_C}{\partial F_A} (C_0 - \tau_{CA}) \right] + \frac{\partial S_B}{\partial F_A} q_B (\tau_{AB} - C_0) + \frac{\partial S_C}{\partial F_A} q_C (\tau_{AC} - C_0) + 1 \right\} = 0 \end{aligned} \quad (A2)$$

Note $\frac{\partial S_A}{\partial F_A} = \frac{x_1 - 1}{t}$, $\frac{\partial S_B}{\partial F_A} = \frac{x_3}{t}$ and $\frac{\partial S_C}{\partial F_A} = \frac{x_2}{t}$. Dividing Eq (A1) by $\frac{\partial S_A}{\partial P_A}$ yields,

$$\begin{aligned} \{q_A [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0) + F_A - f\} + S_A / \frac{\partial S_A}{\partial P_A} \cdot \left\{ \frac{\partial q_A}{\partial P_A} [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + q_A \left[1 + \frac{\partial S_B}{\partial P_A} (C_0 - \tau_{BA}) + \frac{\partial S_C}{\partial P_A} (C_0 - \tau_{CA}) \right] + \frac{\partial S_B}{\partial P_A} q_B (\tau_{AB} - C_0) + \frac{\partial S_C}{\partial P_A} (\tau_{AC} - C_0) \right\} = 0 \end{aligned} \quad (A3)$$

Also note that $\frac{\partial S_A}{\partial P_A} = q_A \frac{x_1 - 1}{t}$, $\frac{\partial S_B}{\partial P_A} = q_A \frac{x_3}{t}$ and $\frac{\partial S_C}{\partial P_A} = q_A \frac{x_2}{t}$ (the fact that the utility is measured in monetary terms implies that $\frac{\partial w_A}{\partial P_A} = -q_A$ in view of Roy's lemma). Dividing Eq (A2) by $\frac{\partial S_A}{\partial F_A}$ yields,

$$\begin{aligned} \{q_A [P_A - 2C_0 + S_B(C_0 - \tau_{BA}) + S_C(C_0 - \tau_{CA})] + S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0) + F_A - f\} + \frac{S_A}{\frac{\partial S_A}{\partial F_A}} + S_A \left\{ q_A \left[\left(-\frac{x_3}{x_2 + x_3} \right) (C_0 - \tau_{BA}) + \left(-\frac{x_2}{x_2 + x_3} \right) (C_0 - \tau_{CA}) \right] + q_B (\tau_{AB} - C_0) \left(-\frac{x_3}{x_2 + x_3} \right) + q_C (\tau_{AC} - C_0) \left(-\frac{x_2}{x_2 + x_3} \right) \right\} = 0 \end{aligned} \quad (A4)$$

Combining Eqs (A3) and (A4), we obtain,

$$\frac{S_A}{\partial S_A / \partial P_A} \cdot \frac{\partial q_A}{\partial P_A} [S_A(P_A - 2C_0) + S_B(P_A - C_0 - \tau_{BA}) + S_C(P_A - C_0 - \tau_{CA})] = 0$$

Hence,

$$P_A = S_A(2C_0) + S_B(\tau_{BA} + C_0) + S_C(\tau_{CA} + C_0) \quad (\text{A5})$$

Making use of formula (A5), we obtain the following from (A4),

$$F_A = f + \frac{tS_A}{x_2+x_3} + \frac{x_3}{x_2+x_3} S_A q_B (\tau_{AB} - C_0) + \frac{x_2}{x_2+x_3} S_A q_C (\tau_{AC} - C_0) - [S_B q_B (\tau_{AB} - C_0) + S_C q_C (\tau_{AC} - C_0)] + S_A q_A \left[\left(\frac{x_3}{x_2+x_3} \right) (C_0 - \tau_{BA}) + \left(\frac{x_2}{x_2+x_3} \right) (C_0 - \tau_{CA}) \right] \quad (\text{A6})$$

Step 2, the existence of the equilibrium. We first introduce a lemma.

Lemma A1. For a given F_A , there exists a unique corresponding P_A that maximizes firm A's profit.

The proof of Lemma A1. Consider the perceived marginal pricing rule of the per-minute call, as formulated in (3.5A), or (A5). Noting that $\frac{\partial S_A}{\partial P_A} = q_A \frac{x_1-1}{t}$, $\frac{\partial S_B}{\partial P_A} = q_A \frac{x_3}{t}$ and $\frac{\partial S_C}{\partial P_A} = q_A \frac{x_2}{t}$, one obtains the following. For the LHS of Eq (3.5A), $\frac{\partial P_A}{\partial P_A} = 1$. For the RHS of Eq (3.5A), $\frac{\partial(RHS)}{\partial P_A} = \frac{q_A}{t} [x_3(\tau_{BA} - C_0) + x_2(\tau_{CA} - C_0)] \approx 0$, for a sufficiently large t or for access charges close to C_0 . Hence, allowing for changes in P_A only and fixing the other prices, there exists one unique solution to Eq (3.5A). Lemma A1 is thus established.

Now turn to the optimal solution of F_A (note that for a given value of F_A , there exists a unique $P_A(F_A)$ as shown above). From the profit function Eq.(3.5C), the FOC is,

$$\frac{\partial \pi_A}{\partial F_A} = \frac{\partial(S_A S_B)}{\partial F_A} q_B (\tau_{AB} - C_0) + \frac{\partial(S_A S_C)}{\partial F_A} q_C (\tau_{AC} - C_0) + S_A + \frac{\partial S_A}{\partial F_A} (F_A - f) = 0. \text{ In light of } \frac{\partial S_A}{\partial F_A} = \frac{x_1-1}{t}, \frac{\partial S_B}{\partial F_A} = \frac{x_3}{t} \text{ and } \frac{\partial S_C}{\partial F_A} = \frac{x_2}{t}, \text{ we have}$$

$$\frac{\partial \pi_A}{\partial F_A} = \left[S_A \cdot \frac{x_3}{t} + S_B \cdot \frac{x_1-1}{t} \right] q_B (\tau_{AB} - C_0) + \left[S_A \cdot \frac{x_2}{t} + S_C \cdot \frac{x_1-1}{t} \right] q_C (\tau_{AC} - C_0) + S_A + (F_A - f) \cdot \frac{x_1-1}{t} = 0 \quad \dots\dots\dots(\text{A7})$$

The SOC can thus be obtained in view of the fact that $\frac{\partial x_1}{\partial F_A} = \frac{2}{3t}$ and $\frac{\partial x_2}{\partial F_A} = \frac{\partial x_3}{\partial F_A} = -\frac{1}{3t}$,

$$\frac{\partial^2 \pi_A}{\partial F_A^2} = [s_A \cdot (-\frac{1}{3t^2}) + s_B \cdot \frac{2}{3t^2} + \frac{2(x_1-1)x_3}{t^2}]q_B(\tau_{AB} - C_0) + [s_A \cdot (-\frac{1}{3t^2}) + s_C \cdot \frac{2}{3t^2} + \frac{2(x_1-1)x_2}{t^2}]q_C(\tau_{AC} - C_0) + \frac{2(x_1-1)}{t} + (F_A - f) \cdot \frac{2}{3t^2} \quad (\text{A8})$$

Thus, when t is sufficiently large, $\text{sign}\left(\frac{\partial^2 \pi_A}{\partial F_A^2}\right) = \text{sign}\left(\frac{2(x_1-1)}{t}\right) = -1$. Or, when access charges are all close to C_0 , SOC, equation (A8), becomes $\frac{\partial^2 \pi_A}{\partial F_A^2} \approx \frac{2(x_1-1)}{t} + (F_A - f) \cdot \frac{2}{3t^2}$. But FOC then turns out to be, $S_A + (F_A - f) \cdot \frac{x_1-1}{t} \approx 0$. Hence, $\frac{\partial^2 \pi_A}{\partial F_A^2} \approx \frac{2}{3t(1-x_1)} [S_A - 3(1-x_1)^2]$. But it can be easily shown from Lemma 2 that $S_A \leq \frac{3(1-x_1)^2}{4}$. Hence, $\frac{\partial^2 \pi_A}{\partial F_A^2} \leq -\frac{3(1-x_1)}{2} < 0$ for access charges all being close to C_0 .

Thus, when t is sufficiently large, or the access charges are all close to C_0 , for any given values of (F_B, F_C) , there exists a unique F_A , denoted as F_A^* , that maximizes network A's profit. By Berge's lemma, F_A^* is continuous in (F_B, F_C) . On the other hand, by (A5), $P_A \geq C_0$. Similar reasoning leads to $P_B, P_C \geq C_0$. Consequently, both q_B and q_C are upper bounded. Hence, by (A7), there exists a certain large number M such that $F_A^* \leq f + M$.¹⁷ Define $H(F_A, F_B, F_C) = (F_A^*, F_B^*, F_C^*)$. Then, H is a continuous mapping from $[0, f + M]^3$ into itself. By Brouwer's theorem, there exists a fixed point (Nash equilibrium) for H .

Step 3, the uniqueness of the equilibrium. The basic construct of our argument is a modified version of the method developed in Laffont et al (1998a) that establishes the uniqueness of equilibrium by considering the slopes of the ‘‘pseudo reaction curves’’. The major problem in doing so for our model, however, is that there does not exist the slope of the reaction curve (in response to two offers, rather than one). Nonetheless, as shown below, we may reduce the interaction between three offers, both formally and genuinely, into one between two pseudo reaction curves.

When every access charge is equal to C_0 , from formula (3.5B),

$$S_A + (F_A - f) \frac{x_1-1}{t} = 0 \quad (\text{A9})$$

¹⁷ A careful reader may be concerned with the possibility of $(x_1 - 1)$ approaching zero. But that does not constitute a problem, for, by Lemma 2, we obtain after some algebraic manipulation that $S_A \in [0, \frac{3}{4}(1-x_1)^2]$. Thus, if $x_1 - 1$ does approaches zero the last term on RHS of (A7) goes to zero no faster than the other terms. To illustrate this point, one may consider, for instance, the scenario when all the access charges are equal to the traffic cost. It follows from (A7), $\frac{\partial \pi_A}{\partial F_A} \geq 0$ if and only if $F_A - f \leq \frac{tS_A}{1-x_1}$. But by Lemma 2, $S_A \leq \frac{3}{4}(1-x_1)^2$. Hence, $F_A - f \leq \frac{3t}{4}$.

$$s_B + (F_B - f) \frac{x_2^{-1}}{t} = 0 \quad (\text{A10})$$

$$s_C + (F_C - f) \frac{x_3^{-1}}{t} = 0 \quad (\text{A11})$$

Note equation (A9) describes the reaction of firm A, F_A , to the price strategies of firms B and C, (F_B, F_C) . The LHS of (A9) decreases in F_A . That is, for given (F_B, F_C) , there exists a unique F_A that maximizes firm A's profit. Similarly, (A10) and (A11) respectively represent the reactions of firms B and C. Total differentiation of each of the three equations, in view of the fact that $\frac{\partial s_A}{\partial F_A} = \frac{x_1 - 1}{t}$,

$\frac{\partial s_B}{\partial F_A} = \frac{x_3}{t}$ and $\frac{\partial s_C}{\partial F_A} = \frac{x_2}{t}$ and the fact that $\frac{\partial x_1}{\partial F_A} = \frac{2}{3t}$ and $\frac{\partial x_2}{\partial F_A} = \frac{\partial x_3}{\partial F_A} = -\frac{1}{3t}$, yields,

$$\frac{2(x_1 - 1)}{t} dF_A + \frac{x_3}{t} dF_B + \frac{x_2}{t} dF_C + \frac{F_A - f}{3t^2} (2dF_A - dF_B - dF_C) = 0 \quad (\text{A12})$$

$$\frac{2(x_2 - 1)}{t} dF_B + \frac{x_3}{t} dF_A + \frac{x_1}{t} dF_C + \frac{F_B - f}{3t^2} (2dF_B - dF_A - dF_C) = 0 \quad (\text{A13})$$

$$\frac{2(x_3 - 1)}{t} dF_C + \frac{x_1}{t} dF_B + \frac{x_2}{t} dF_A + \frac{F_C - f}{3t^2} (2dF_C - dF_A - dF_B) = 0 \quad (\text{A14})$$

Recall that formula (A12) (or formula (A9)) describes how firm A reacts to firms B and C's strategies. Combining (A12) and (A13) then leads to the reaction of firm A, in taking into account firm B's reaction to firm C's strategy as well as to firm A's strategy, to firm C's offer, essentially a mapping from F_C to F_A , denoted as $F_A(F_C)$. It can be shown that (A12) implies $\frac{F_A - f}{3t} + (x_1 - 1) < 0$.

Also note that it follows from (A9) and Lemma 2 that $\frac{F_A - f}{3t} - x_3 \approx \frac{S_A}{3(1 - x_1)} - x_3 = \frac{x_2^2 - 2x_2x_3 - 5x_3^2}{6(1 - x_1)}$ and

$\frac{F_A - f}{3t} - x_2 \approx \frac{x_3^2 - 2x_2x_3 - 5x_2^2}{6(1 - x_1)}$. Thus, if the relative advantages of the three networks are not too

large, then the values of x_1 , x_2 and x_3 are not too different, resulting in $\frac{F_A - f}{3t} - x_3 < 0$ and

$$\frac{F_A - f}{3t} - x_2 < 0.$$

We now rewrite (A12) as,

$$dF_A = \rho_1 dF_B + \rho_2 dF_C \text{ where } \rho_1 = \frac{\frac{F_A - f}{3t} x_3}{\frac{2(F_A - f)}{3t} + 2(x_1 - 1)}, \text{ and } \rho_2 = \frac{\frac{F_A - f}{3t} x_2}{\frac{2(F_A - f)}{3t} + 2(x_1 - 1)} \quad (\text{A15})$$

Then $\rho_1, \rho_2 > 0$. Further note that $\rho_1 + \rho_2 = \frac{\frac{2(F_A - f)}{3t} + (x_1 - 1)}{\frac{2(F_A - f)}{3t} + 2(x_1 - 1)} = \frac{1}{2} + \frac{\frac{F_A - f}{3t}}{\frac{2(F_A - f)}{3t} + 2(x_1 - 1)} < \frac{1}{2}$.

In a similar vein, we may put (A13) in the form

$$dF_B = \rho_3 dF_A + \rho_4 dF_C \quad (\text{A16})$$

for appropriately defined terms, ρ_3 and ρ_4 , and show that $\rho_3, \rho_4 > 0$ and $\rho_3 + \rho_4 < \frac{1}{2}$. Substituting

(A16) into (A15) yields, $\frac{dF_A}{dF_C} = \frac{\rho_1 \rho_4 + \rho_2}{1 - \rho_1 \rho_3} < \frac{\rho_1 \left(\frac{1}{2} - \rho_3\right) + \frac{1}{2} - \rho_1}{1 - \rho_1 \rho_3} < \frac{1}{2}$. The same argument applied to network

C's pseudo reaction curve $F_C(F_A)$, constituted in taking account of firm B's strategic reaction, leads

us to $\frac{dF_C}{dF_A} < \frac{1}{2}$. Thus, the two positive-sloped pseudo reaction curves $F_A(F_C)$ and $F_C(F_A)$ have at most

one intersection (Laffont et al 1998a, de Bijl and Peitz 2002, pp. 78-81).

When t is sufficiently large, one can easily show the uniqueness of the intersection of a similarly constructed pseudo reactions curves. We omit the algebraic details to save space. **QED**

Appendix B.

The proof of Lemma 4. We proceed by showing that the coordinates of the agent who is indifferent to the three networks, which determine the market shares of the networks, remain unchanged when the uniform access charge changes marginally around the marginal cost C_0 . For this purpose, we need to first consider how the indifferent individual agent's surplus (net surplus plus the transportation cost) w_i ($i = A, B$ or C) changes in the access charge around C_0 . Note $\frac{\partial w_A}{\partial P_A} = -q_A$ by

Roy's lemma. One thus obtains from (2.2), (3.5A) and (3.5B) by some algebraic manipulation, $\frac{\partial w_A}{\partial P_A} = -q_A \frac{\partial P_A}{\partial \tau} - \frac{\partial F_A}{\partial \tau} = -\frac{\partial}{\partial \tau} \left(\frac{tS_A}{x_2 + x_3} \right)$ when $\tau = C_0$, at which by (3.5A) all the one-minute prices P_i

($i = A, B$ or C) equal $2C_0$ and hence $q_A = q_B = q_C$. But by Lemma 2, $\frac{S_A}{x_2 + x_3} = \frac{1-x_1}{2} + x_2 - \frac{x_2^2}{1-x_1}$,

$\frac{S_B}{x_1 + x_3} = \frac{1-x_2}{2} + x_1 - \frac{x_1^2}{1-x_2}$ and $\frac{S_C}{x_1 + x_2} = \frac{x_1 + x_2}{2} + \frac{x_1 x_2}{x_1 + x_2}$. Then, as a consequence, we obtain by

manipulating the algebra from Lemma 1 that,

$$\left[\frac{11}{2} - \frac{2x_1}{1-x_2} + \frac{2x_2^2}{(1-x_1)^2} + \frac{x_2^2}{(x_1+x_2)^2} \right] x_1' = \left[2 - \frac{4x_2}{1-x_1} + \frac{x_1^2}{(1-x_2)^2} - \frac{x_1^2}{(x_1+x_2)^2} \right] x_2' \quad (\text{B1})$$

where $x_i' \equiv \left(\frac{\partial x_i}{\partial \tau} \right)_{\tau=C_0}$, $i=1,2$.

By symmetry, we must also have,

$$\left[\frac{11}{2} - \frac{2x_2}{1-x_1} + \frac{2x_1^2}{(1-x_2)^2} + \frac{x_1^2}{(x_1+x_2)^2} \right] x_2' = \left[2 - \frac{4x_1}{1-x_2} + \frac{x_2^2}{(1-x_1)^2} - \frac{x_2^2}{(x_1+x_2)^2} \right] x_1' \quad (\text{B2})$$

It follows from (B1) and (B2) that $H(x_1, x_2)x_1' = 0$ where $H(x_1, x_2) \equiv$

$$\left[\frac{11}{2} - \frac{2x_1}{1-x_2} + \frac{2x_2^2}{(1-x_1)^2} + \frac{x_2^2}{(x_1+x_2)^2} \right] \left[\frac{11}{2} - \frac{2x_2}{1-x_1} + \frac{2x_1^2}{(1-x_2)^2} + \frac{x_1^2}{(x_1+x_2)^2} \right] - \left[2 - \frac{4x_2}{1-x_1} + \frac{x_1^2}{(1-x_2)^2} - \frac{x_1^2}{(x_1+x_2)^2} \right] \left[2 - \frac{4x_1}{1-x_2} + \frac{x_2^2}{(1-x_1)^2} - \frac{x_2^2}{(x_1+x_2)^2} \right].$$

It can be shown that $H(x_1, x_2) > 0$ for any $x_1, x_2 > 0$ such that $x_1 + x_2 < 1$ (to save space we omit the detailed computation, which is available upon request from the authors). Thus, $x_1' = 0$. As a consequence, $x_2' = 0$. Hence $x_3' = 0$. That is, the market shares remain unchanged when the cost-based uniform access charge changes marginally. **QED**

The proof of Proposition 2. The social welfare under the uniform access charge regime is denoted as E_U and defined as the sum of the consumer surplus and the three firms' profits. We have,

$$E_U = \sum_{i \in \{A, B, C\}} S_i [v(P_i) + \theta_i + v_0] + \sum_{i \in \{A, B, C\}} S_i \sum_{j \neq i} S_j q_j (\tau - C_0) - f - \frac{t}{2} (1 - 6x_1 x_2 x_3) \quad (\text{B3})$$

Note that $\frac{\partial v(P_i)}{\partial \tau} = -q_i \sum_{j \neq i} S_j$ as is shown above already. Then, by Lemma 4,

$$\begin{aligned} \frac{\partial E_U}{\partial \tau} &= \sum_i [S_i' v(P_i) + S_i' \theta_i - q_i S_i \sum_{j \neq i} S_j] + \sum_i S_i' \sum_{j \neq i} S_j q_j (\tau - C_0) + \sum_i S_i \sum_{j \neq i} S_j' q_j (\tau - C_0) + \\ &\sum_i S_i \sum_{j \neq i} S_j \frac{\partial q_j}{\partial P_j} (1 - S_j) (\tau - C_0) + \sum_i S_i \sum_{j \neq i} S_j q_j + 3t \frac{\partial (x_1 x_2 x_3)}{\partial \tau} = 0 \end{aligned}$$

at $\tau = C_0$, where $S_i' \equiv \frac{\partial S_i}{\partial \tau}$, $i=1,2,3$.

We now consider the SOC at $\tau = C_0$. Note $P_i = 2C_0$, for any network $i, i \in \{A, B, C\}$ at the cost-based access charge. By Lemma 4 again,

$$\begin{aligned} &\frac{\partial^2 E_U}{\partial \tau^2} \\ &= \sum_i [S_i'' v(P_i) + S_i'' \theta_i] + \sum_i S_i \sum_{j \neq i} S_j (1 - S_j) \frac{\partial q_j}{\partial P_j} + 3t \frac{\partial^2 (x_1 x_2 x_3)}{\partial \tau^2} \\ &= \sum_i S_i'' (\theta_i - \bar{\theta}) + \sum_i S_i \sum_{j \neq i} S_j (1 - S_j) \frac{\partial q_j}{\partial P_j} + 3t \frac{\partial^2 (x_1 x_2 x_3)}{\partial \tau^2} \end{aligned} \quad (\text{B4})$$

where $\bar{\theta} \equiv \frac{1}{3} \sum_i \theta_i$, and $S_i'' \equiv \left(\frac{\partial^2 S_i}{\partial \tau^2} \right)_{\tau=C_0}$. When the three networks have no relative advantage with each other in terms of appeal of their services to the consumers, i.e. $\theta_A = \theta_B = \theta_C$, the value of $x_1 x_2 x_3$ remains a constant $(1/3^3)$. As a consequence, the third term on RHS of (B4), as well as the first term, vanishes. Hence, $\frac{\partial^2 E_U}{\partial \tau^2} = \sum_i S_i \sum_{j \neq i} S_j (1 - S_j) \frac{\partial q_j}{\partial P_j} < 0$. By continuity, when the value of the three parameters $(\theta_A, \theta_B, \theta_C)$ do not significantly differ from one another, the sum of the first and

third terms is dominated by the second term, since $P_i = 2C_0$ for any network i at the cost-based access charge and hence $\frac{\partial q_i}{\partial P_i}$ remains a *negative constant*, independent of both the firm-specificity i and changes in the parameters. Thus, $\frac{\partial^2 E_U}{\partial \tau^2} < 0$. **QED**

The proof of Lemma 5. The proof is somewhat similar to that of Lemma 4. When all the access charges are equal C_0 , all the one-minute prices, by (3.5A), equal $2C_0$, and hence $q_A = q_B = q_C$. We thus obtain from (2.2), (3.5A) and (3.5B) by some algebraic manipulation, $\frac{\partial w_A}{\partial \tau_{AB}} = -q_A S_B - \frac{\partial F_A}{\partial \tau_{AB}}$
 $= -\frac{\partial}{\partial \tau_{AB}} \left(\frac{tS_A}{1-x_1} \right)$. By similar reasoning to that in the proof of Lemma 4, we can obtain two equations, which are formally the same as (B1) and (B2) save replacing $\frac{\partial x_i}{\partial \tau}$ with $\frac{\partial x_i}{\partial \tau_{AB}}$, ($i=1,2$), and from which we can therefore show that $\left(\frac{\partial x_1}{\partial \tau_{AB}} \right)_{\tau_{AB}=C_0} = \left(\frac{\partial x_2}{\partial \tau_{AB}} \right)_{\tau_{AB}=C_0} = 0$. **QED**

The proof of Proposition 3. Parallel to the proof of Proposition 2, we denote by E_R the social welfare under the reciprocal access charge regime, which is equal to the sum of the consumer surplus and the three firms' profits. By (3.5C),

$$E_R = \sum_{i \in \{A,B,C\}} S_i [v(P_i) + \theta_i + v_0] + \sum_{i \in \{A,B,C\}} S_i \sum_{j \neq i} S_j q_j (\tau_{ij} - C_0) - f - \frac{t}{2} (1 - 6x_1 x_2 x_3) \quad (\text{B5})$$

Now fix the reciprocal access charge between networks A and C, as well as that between networks B and C, both at the cost basis, i.e., $\tau_{AC} = \tau_{BC} = C_0$, and consider the marginal change in τ_{AB} around C_0 . By equation (3.5A), $\frac{\partial P_A}{\partial \tau_{AB}} = S_B$, $\frac{\partial P_B}{\partial \tau_{AB}} = S_A$, and $\frac{\partial P_C}{\partial \tau_{AB}} = 0$. Thus,

$$\begin{aligned} \frac{\partial v(P_A)}{\partial \tau_{AB}} &= \frac{\partial v(P_A)}{\partial P_A} \cdot \frac{\partial P_A}{\partial \tau_{AB}} = (-q_A) S_B, \quad \frac{\partial v(P_B)}{\partial \tau_{AB}} = (-q_B) S_A, \quad \text{and} \quad \frac{\partial v(P_C)}{\partial \tau_{AB}} = 0. \end{aligned}$$

In view of Lemma 5, at $\tau_{AB} = C_0$,

$$\begin{aligned} \frac{\partial E_R}{\partial \tau_{AB}} &= \sum_{i \in \{A,B,C\}} \frac{\partial S_i}{\partial \tau_{AB}} [v(P_i) + \theta_i] + S_A [(-q_A) S_B] + S_B [(-q_B) S_A] + \frac{\partial (S_A S_B q_B)}{\partial \tau_{AB}} (\tau_{AB} - C_0) + \\ &\frac{\partial (S_A S_B q_A)}{\partial \tau_{AB}} (\tau_{AB} - C_0) + 3t \frac{\partial (x_1 x_2 x_3)}{\partial \tau_{AB}} = 0 \end{aligned} \quad (\text{B6})$$

Now consider the SOC at $\tau_{AB} = C_0$. Note $P_i = 2C_0$, for any network i , $i \in \{A, B, C\}$ at the cost-based access charge. By Lemma 5 again, at $\tau_{AB} = C_0$, $\frac{\partial^2 E_R}{\partial \tau_{AB}^2} = \sum_{i \in \{A,B,C\}} \theta_i \frac{\partial^2 S_i}{\partial \tau_{AB}^2} + S_A S_B \left[\frac{\partial q_A}{\partial \tau_{AB}} + \frac{\partial q_B}{\partial \tau_{AB}} \right] + 3t \frac{\partial^2 (x_1 x_2 x_3)}{\partial \tau_{AB}^2}$. Note $\frac{\partial q_A}{\partial \tau_{AB}} = \frac{\partial q_A}{\partial P_A} \cdot \frac{\partial P_A}{\partial \tau_{AB}} = \frac{\partial q_A}{\partial P_A} S_B$ and $\frac{\partial q_B}{\partial \tau_{AB}} = \frac{\partial q_B}{\partial P_B} S_A$. Hence, $\frac{\partial^2 E_R}{\partial \tau_{AB}^2} = \sum_{i \in \{A,B,C\}} \theta_i \frac{\partial^2 S_i}{\partial \tau_{AB}^2} +$

$S_A S_B \left[S_A \frac{\partial q_B}{\partial \tau_{AB}} + S_B \frac{\partial q_A}{\partial \tau_{AB}} \right] + 3t \frac{\partial^2(x_1 x_2 x_3)}{\partial \tau_{AB}^2}$. Note $P_A = P_B = 2C_0$ at $\tau_{ij} = C_0, \forall i, j \in \{A, B, C\}, i \neq j$, and consequently $\frac{\partial q_i}{\partial P_i}$ remains a *negative constant*, independent of both the firm-specificity i and changes in the parameters. By similar reasoning to that in the proof of Proposition 2, one can thus show from (B6) that $\frac{\partial^2 E_R}{\partial \tau_{AB}^2} < 0$, provided the three networks do not have significant relative advantage with each other in terms of appeal of their services to the consumers. **QED**

Appendix C.

The proof of Lemma 6. (i). Applying the same technique used in the proof of Lemmas 4 and 5, we analyze the change in the coordinates of the indifferent agent, caused by a marginal change of τ_1 from the marginal cost C_0 when $\tau_2 = \tau_3 = C_0$. For this purpose, we first consider how the indifferent individual agent's surplus w_i ($i = A, B$ or C) may change. Since firms B and C are identical, $w_B = w_C$ and $x_2 = x_3$. Also note that at the cost based access charges $q_A = q_B = q_C$, denoted as q for notational neatness. We can obtain from (2.2), (3.5A) and (3.5B) that

$$\left. \frac{w_A}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} = -\frac{\partial}{\partial \tau_1} \left(\frac{tS_A}{x_2+x_3} \right) - (S_A - S_B - S_C)q \quad (C1)$$

$$\left. \frac{w_B}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} = -S_A q - \frac{\partial}{\partial \tau_1} \left(\frac{tS_B}{x_1+x_3} \right) + \frac{x_3}{x_1+x_3} S_B q \quad (C2)$$

By Lemma 1, we obtain in view of $x_2 = x_3$ that, $\left. \frac{\partial x_2}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} = \frac{1}{3t} \left. \frac{\partial(w_A - w_B)}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0}$. Making

use of (C1) and (C2), $\left. \frac{\partial x_2}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} = \left[\frac{t}{(1-x_2)^2} + 3t \right]^{-1} \left(1 + \frac{x_1}{x_1+x_2} \right) S_B q > 0$. Hence, by Lemma 2,

$$\left. \frac{\partial S_A}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} > 0.$$

(ii). By quite similar reasoning to that in the proof of part (i), one obtains (details omitted to save

space), $\left. \frac{\partial x_2}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0} = - \left[\frac{t}{(1-x_2)^2} + 3t \right]^{-1} \left(1 + \frac{x_1}{x_1+x_2} \right) S_B q < 0$, and therefore, $\left. \frac{\partial S_B}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0} =$

$$\left. \frac{\partial S_C}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0} > 0.$$

(iii) Quite similar to the above reasoning, the change in the coordinates of the indifferent agent induced by local variation of τ_3 around the cost basis, can be derived as

$$\left. \frac{\partial x_2}{\partial \tau_3} \right|_{\tau_1=\tau_2=\tau_3=C_0} = 0. \text{ Hence, } \left. \frac{\partial S_i}{\partial \tau_3} \right|_{\tau_1=\tau_2=\tau_3=C_0} = 0, \quad i = A, B, C. \quad \mathbf{QED}$$

The proof of Proposition 4. We first formulate the social welfare, denoted as E_{1-t0-2}^D to represent the discretionary regime under the one-incumbent-and-two-entrants scenario,

$$E_{1-t0-2}^D = S_A[v(P_A) + \theta_A] + 2S_B[v(P_B) + \theta_B] + v_0 + 2S_A S_B (\tau_1 - C_0) q_B + 2S_B^2 (\tau_3 - C_0) q_B + 2S_B S_A (\tau_2 - C_0) q_A - f - \frac{t}{2} (1 - 6x_1 x_2^2) \quad (C3)$$

We have used the fact $w_B = w_C$ and $x_2 = x_3$ in formulating E_{1-t0-2}^D in the above. Also note that at the cost based access charges $q_A = q_B = q_C$, denoted as q for notational neatness.

When τ_1 marginally changes around C_0 while $\tau_2 = \tau_3 = C_0$, by Lemma 1, lemma 2 and equation (3.5B),

$$\begin{aligned} \left. \frac{\partial E_{1-t0-2}^D}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} &= (\theta_A - \theta_B) \left. \frac{\partial S_A}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} + 3t \left. \frac{\partial}{\partial \tau_1} [(1 - 2x_2)x_2^2] \right|_{\tau_1=\tau_2=\tau_3=C_0} \\ &= 6x_2 [t(1 - 3x_2) + \theta_A - \theta_B] \left. \frac{\partial x_2}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} \\ &= 6x_2 t \cdot \left. \frac{3x_2-1}{2(1-x_2)} \frac{\partial x_2}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} \end{aligned}$$

For the case of one incumbent firm A, and two entrants, B and C, $x_2 > \frac{1}{3}$. From the established fact in the proof of lemma 6, $\left. \frac{\partial x_2}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0}$, we thus obtain $\left. \frac{\partial E_{1-t0-2}^D}{\partial \tau_1} \right|_{\tau_1=\tau_2=\tau_3=C_0} > 0$.

When τ_2 marginally changes around C_0 while $\tau_1 = \tau_3 = C_0$, by similar reasoning we obtain, $\left. \frac{\partial E_{1-t0-2}^D}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0} = 6x_2 t \cdot \left. \frac{3x_2-1}{2(1-x_2)} \frac{\partial x_2}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0}$. But $\left. \frac{\partial x_2}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0} < 0$ as shown in the proof of Lemma 6, we have, $\left. \frac{\partial E_{1-t0-2}^D}{\partial \tau_2} \right|_{\tau_1=\tau_2=\tau_3=C_0} < 0$.

When τ_3 marginally changes around C_0 while $\tau_1 = \tau_2 = C_0$, it can be shown by quite similar reasoning to the above, $\left. \frac{\partial E_{1-t0-2}^D}{\partial \tau_3} \right|_{\tau_1=\tau_2=\tau_3=C_0} = 6x_2 t \cdot \left. \frac{3x_2-1}{2(1-x_2)} \frac{\partial x_2}{\partial \tau_3} \right|_{\tau_1=\tau_2=\tau_3=C_0} = 0$. **QED**

The proof of Lemma 7. (i). The same as part (i) of Lemma 6.

(ii). By quite similar reasoning to that in the proof of Lemma 6, we can get, $\left. \frac{\partial x_2}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0} = - \left[\frac{t}{(1-x_2)^2} + 3t \right]^{-1} \left(1 + \frac{x_1}{x_1+x_2} \right) S_B q < 0$, and therefore, $\left. \frac{\partial S_B}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0} = \left. \frac{\partial S_C}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0} > 0$. **QED**

The proof of Proposition 5. Under non-discriminatory access charge regime with two identical networks B and C, like in the proof of Lemma 6, we denote by τ_A the incumbent network A's access charge to the two entrants and by τ_B each entrant's access charge to the other firms (due to their identity the same access charge is adopted by the two entrant networks). The social welfare, denoted as $E_{1-t_0-2}^{ND}$ to reflect the non-discriminatory regime being applied to the one-incumbent-and-two-entrants scenario, equals,

$$E_{1-t_0-2}^{ND} = S_A[v(P_A) + \theta_A] + 2S_B[v(P_B) + \theta_B] + v_0 + 2S_A S_B q_B (\tau_A - C_0) + 2S_B [S_A q_A + S_B q_B] (\tau_B - C_0) - f - \frac{t}{2} (1 - 6x_1 x_2^2) \quad (C4)$$

Like in the proof of Lemma 6, we have used the fact $w_B = w_C$ and $x_2 = x_3$ in formulating $E_{1-t_0-2}^{ND}$ in the above. Also note that at the cost based access charges $q_A = q_B = q_C$, denoted as q for notational neatness.

We are now ready to prove Proposition 5.

(i) The FOC of $E_{1-t_0-2}^{ND}$ with respect to τ_A at $\tau_A = \tau_B = C_0$ can be obtained from equation (C4) (some algebraic details are omitted to save space),

$$\begin{aligned} \left. \frac{\partial E_{1-t_0-2}^{ND}}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} &= (\theta_A - \theta_B) \left. \frac{\partial S_A}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} + 3t \left. \frac{\partial}{\partial \tau_A} [(1 - 2x_2)x_2^2] \right|_{\tau_A=\tau_B=C_0} \\ &= 6x_2 [t(1 - 3x_2) + \theta_A - \theta_B] \left. \frac{\partial x_2}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} \end{aligned}$$

Making use of Lemma 1, Lemma 2 and equation

$$(3.5B), \quad \left. \frac{\partial E_{1-t_0-2}^{ND}}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} = 6x_2 [F_A - F_B] \left. \frac{\partial x_2}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} = 6x_2 \left(\frac{tS_A}{x_2+x_3} - \frac{tS_B}{x_1+x_3} \right) \left. \frac{\partial x_2}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} = 6x_2 t \cdot$$

$\frac{3x_2-1}{2(1-x_2)} \left. \frac{\partial x_2}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0}$. For the case of one incumbent firm A, and two entrants, B and C, $x_2 > \frac{1}{3}$. From

the already established fact $\left. \frac{\partial x_2}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} > 0$ we thus obtain $\left. \frac{\partial E_{1-t_0-2}^{ND}}{\partial \tau_A} \right|_{\tau_A=\tau_B=C_0} > 0$.

(ii). Similar reasoning starting from equation (C4) leads to

$$\left. \frac{\partial E_{1-t_0-2}^{ND}}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0} = 6x_2 t \cdot \frac{3x_2-1}{2(1-x_2)} \left. \frac{\partial x_2}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0}. \text{ But } \left. \frac{\partial x_2}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0} < 0 \text{ as shown in the proof of part}$$

(ii) of Lemma 7. Thus, $\left. \frac{\partial E_{1-t_0-2}^{ND}}{\partial \tau_B} \right|_{\tau_A=\tau_B=C_0} < 0$.

QED

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